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Discrete Hartley Transform and its applications - A review

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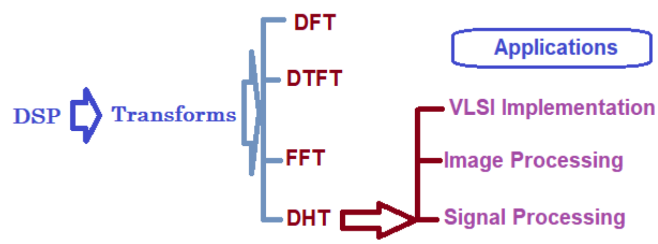
ABSTRACT



Digital signal processing (DSP) has become the single most powerful essential component in all technological applications, which include multimedia as well as mobile communications, data compression, network cameras, mobile phones, sensor imaging, acoustic beam formers, GPS, but also biomedical signal processing, and so on. Almost all of these applications place various demands on DSP systems, such as the ability to handle high throughput data required by real-world applications, reduced power, and the need for chip space. Transforms help convert one data type to another in DSP applications. The Hartley Transform (DHT) is important in Digital Signal Processing. When the input sequence is real, DHT is equivalent to Fast Fourier transform and is utilized in image and optical signal processing, computer vision, teleconferencing, and processing or analysis of moving images. Development of DHT algorithms for application in optical signal processing, image processing, and VLSI implementations have been discussed in this review.

Keywords: Digital Signal Processing, Discrete Fourier Transform, Discrete Hartley Transform, VLSI implementation, Image processing,

INTRODUCTION

Digital Signal Processing (DSP) is a term that refers to data processing in numerous domains depending on its applications. DSP is used in various sectors, including space, medical, economic, industrial, and scientific [1]. Each one necessitates the analysis of large amounts of data to gather relevant information. Transform is indeed a DSP approach for transforming one data type into another. In DSP, there is a transformed family for data processing. One of several oldest techniques within that family includes Fourier analysis [2]. The Fourier transform is indeed an integral-based mathematical technique. On the other hand, the Fourier transform is ineffective for non-stationary waveforms. Because neither the Fourier series nor the Fourier transform applies to discrete signals, a unique discrete spectrum transform is required. Regarding signals that range from positive to negative infinite and are not periodic, the discrete-time Fourier transform (DTFT) is utilized [3]. Because DTFT isn't suitable for periodic discrete signals, the discrete Fourier transform (DFT) was created. DFT is still a discrete numerical version of FT that uses accumulation rather than integrals. Signals which repeat consecutively in a periodic pattern ranging from positive to negative infinity are treated with DFT. FFT is a DFT upgrade in which the calculation is faster [4]. For a long time, discrete transformations with low multiplicative complexity have attracted the curiosity of researchers [5]. DHT is an acronym for Discrete Hartley Transform, which R. V. L. Hartley introduced in 1942. Discrete Hartley Transform is invented to reduce this hardware complexity. It seems to be a Radix-2 method that is particularly effective in Signal processing applications, including image compression, signal shrinkage, filter banks, signalling representation, including harmonic analysis, among others [6]. The usage of DHT is primarily intended to simplify the intricacy of the VLSI hardware platform. Although DHT computing has also advanced in hardware complexity, it is becoming less. It is because the forward and inverse of DHT are identical except for the scaling factor, or even that DHT is indeed the inverse of itself. Both forward and inverse measurements will be performed on the same hardware. In other words, researchers can presume that both forward and inverse implementations have the same number of adders, multipliers, and other components. The Discrete Fourier Transform (DFT) is comparable to DHT, or DHT is developed from DFT. This DFT contains both real and imaginary components, whereas the DHT only contains real components. Because the computation of imaginary portions is trivial, difficult arithmetic is eliminated. Whenever the input sequence is real, discrete Hartley transform (DHT) would be applied to substitute the DFT [7] efficiently. There are already various low-arithmetic-cost split-radix techniques for computing DHT. Owing to its uneven

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computational structure and the notion that butterflies vary dramatically from stage to stage, this conventional split-radix technique is challenging to implement onto VLSI. As a result, it's vital to come up with new parallel VLSI-compatible algorithms [8]. The discrete Hartley transform (DHT) has been utilized for many years in various applications, including communication and signal processing. It gives a simple way to compute circular convolution, correlations, and de-convolution efficiently. Adaptive filtering, data compression, and error control coding are also made easier. DHT's popularity stems from the fact that it features a real-valued and symmetric transform kernel equivalent to the inverse transform. DHT simplifies the computation of DFT and common unified transforms such as the discrete cosine transform (DCT) and discrete Sine transform (DST). The butterfly was already employed in DHT, which has various terminology of multipliers and coefficients [9] since it is important to partition the data into distinct stages only with the aid of the butterfly, similar to FFT. The fundamental difference between it and the DFT is that it converts real inputs into real outputs without involving complex quantities. The DHT can be calculated using DFT and likewise. It necessitates data breakdown into stages that used a butterfly makes great sense to FFT. However, in terms of coefficients but rather multipliers, the butterfly utilized in DHT is significantly different, and the block diagram is described in figure 1. Its number of coefficients increases in lockstep with the length of a DHT sequence. Let N≥4 be a power of two. For any real input sequence $\{\boldsymbol{x(i)}: \boldsymbol{i} = \boldsymbol{0}, \boldsymbol{1}, \boldsymbol{2} \ldots \ldots, \boldsymbol{N-1}\}$.

$$X(k) = DHT(N)\{x(i)\}$$

$$= \sum_{i=0}^{N-1} x(i).cas[2ki\frac{\pi}{N}] \ for\ k = 0,1 \ldots, N-1$$

Where $cas(x) = \cos(x) + \sin(x)$

Figure 1. Block Diagram of DHT

Furthermore, hardware complexity is decreased by sharing multipliers with the same constant while employing the sub-expression sharing approach. This study provides a technical overview of the relevance of the Discrete Hartley transform for efficient VLSI implementation signal processing and image processing, as well as the shortcomings of current DHT methods.

The rest of the paper is organized as follows: section 2 explains the existing techniques based on applications as well as DHT Algorithms which is used for VLSI implementation, Section 3 provides the summary of this paper, Section 4 concludes this work, and also Section 5, gives the references of this work.

LITERATURE SURVEY

In audio or image processing, a DHT is commonly employed. Higher-order DHTs are employed in orthogonal frequency-division multiplexing systems, whereas the four-point DHT is utilized in video coding. The major focus of this literature review is now on image and signal processing with the Discrete Hartley Transform and DHT for VLSI implementation.

DHT Algorithms for VLSI Implementation

Proposed methods must be utilized to obtain a good/efficient VLSI implementation to have an efficient hardware implementation. Dedicated algorithms can be obtained in two ways: by generating new ones or reorganizing existing services to fit necessity [10]. It is important to consider the signal characteristics moves in a VLSI algorithm throughout to get an effective VLSI algorithm. That was an important factor when creating an effective algorithm with such a high-performance VLSI implementation. Thereby, using regular and modular computational frameworks such as cycle convolution as well as circular correlation with local and regular transmitting data is very helpful in acquiring a great VLSI implementation for DCT, IDCT, DST but also IDST, discrete Fourier transform (DFT), or discrete Hartley transforms (DHT) either using systolic array architecture and design paradigm or dispersed arithmetic [11]. Systolic algorithms produce the finest VLSI implementations employing pipeline designs with fairly short length DHT. The context of the prevailing DHT algorithms may not have an effective pipeline implementation due to the so-called retrograde indexing.

Generating different algorithms for such a pipeline VLSI implementation is important, enabling fast DHT algorithms. Dual Split-Radix DHT is a new split-radix method with high structural qualities like flexibility, consistency, and a simplified Signal Flow Graph (SFG) that eliminates retrograde indexing that permits a cost-effective pipeline implementation using DHT [12]. Also, short pseudo and band convolution algorithms were developed to achieve the best VLSI implementation, but VLSI design may be achieved with high parallelism using the DHT method. Furthermore, hardware complexity is decreased by sharing multipliers with the same constant while employing the sub-expression sharing approach. The following table lists some of the available DHT algorithms and the benefits and drawbacks of employing this DHT for VLSI implementations. Some of the existing VLSI implementation algorithms and their advantages, as well as limitations, are described in the table 1.

When compared to other VLSI implementation approaches, the DHT algorithm performs better. VLSI architecture associated with high parallelism may be achieved using the DHT method. Furthermore, by sharing multipliers with the same constant and employing the sub-expression sharing approach, hardware complexity and power consumption decrease. The next section of the literature survey explains a brief survey on DHT algorithms for image processing.

DHT in Image Processing

The authors presented several new image processing algorithms based on the Discrete Hartley Transform, which are detailed below.

Table 1: Survey on DHT algorithms for VLSI implementation

Technique	Advantages	Limitations	Ref.
Split Radix algorithm	Less hardware complexness and multiplier quantity	Due to their uneven computational structure and retrograde indexing, they are challenging to implement.	[6]
Discrete Hartley Transform algorithm	Less VLSI architecture complexity, less power consumption	Due to their asymmetric computational structure and reverse indexing, they are challenging to implement.	[7]
VLSI DHT algorithm	Less multiplier usage and less delay	Requires more space	[8]
Parallel and Pipeline structure of DHT algorithm	Reduce the latency, hardware utilization as well as power consumption	Compared to the pipeline structure, the parallel structure has more on-chip power.	[9]
DST is based on band-correlation structures	High data transmission speed as well as low I/O cost	gives non-zero outputs in place of zero-valued inputs, which influences the mean relative error.	[10]
Systolic array algorithm based on pseudo-band correlation	low hardware and I/O cost with high-speed performances	Less efficient compared to other techniques	[11]
Dual Split-Radix DHT algorithm	Avoids retrograde indexing and efficient hardware implementation	Occupy more space	[12]
Efficient VLSI algorithm based on band-convolution	High computing speed with lower hardware complexity and low I/O cost	Requires a large amount of storage and processing complexity	[13]
VLSI implementation using Short Pseudo cycle convolution	High throughput, low hardware and I/O complexity, and a simple control structure.	High arithmetic complexity	[14]

Huo-sheng et al. [15] introduced a novel multi-image encryption system that relies on the quaternion discrete fractional Hartley transform (QDFrHT) plus enhanced pixel adaptive diffusion, which might also boost encryption capacity while lowering key utilization. A QDFrHT is introduced in this technique to generalize the discrete fractional Hartley transform towards the quaternion transform domain before using it for multi-image encryption. Using the discrete cosine transform (DCT) with Zigzag operations, these original pictures are compressed into four merging images, which are then expressed as quaternion algebra. The proposed QDFrHT and a double random phase encoding approach are then used to handle the quaternion signals. The plaintext-related pixel responsive diffusion, the pixel scrambled experiment and control of stochastic processes are used to obtain the required encryption image, which improves the algorithm's reliability. By adopting an enhanced diffusion mechanism, this proposed cryptosystem maintains the sensitivity of a cryptosystem towards plaintext. In contrast, the identification of the secret keys is free of plaintext, unlike classic cryptosystems whose secret keys rely on the plaintext.

Xueting et al. [16] proposed a spatial domain-based blind watermarking system of colour pictures, including multilayer discrete Hartley transform (DHT), to safeguard the copyrights of colour digital photos inside the current 5G distributed system. The colour watermark image shuffled by Interpolation transform has been straightforwardly concealed into a colour carrier image by analytical analysis in the spatial domain, instead of the complex DHT domain, based on the unique spatial computing property of the peak power coefficient of DHT as well as the energy agglomeration principle of multilevel image transform. Several simulated results show that the proposed approach is not only undetectable and resilient but also secure and cost-effective, and this can effectively safeguard digital copyright.

Kasban et al. [17] proposed a hybrid picture watermarking approach based on the Karhunen Loeve transform (KLT) and the discrete Hartley transform (DHT). The proposed system (KL–DHT-based watermarking scheme) divides the original image into blocks utilizing a spiral scan, calculates the DHT for every block, and then applies the KLT to every DHT block. And during KLT conversion, the image watermarking procedure takes place. Several picture assaults, such as JPEG image compressing, image rotation, image scaling, introducing noise (Gaussian, impulsive, or speckle), and image contrast alteration utilizing adaptive histogram equalization, are used to distort a watermarked image during the watermark extraction process. Moreover, the recommended scheme is implemented using only a block-by-block image scan, as well as the values are presented to those acquired to use other watermarking strategies, including the discrete cosine transform/singular occur as a result (DCT–SVD) based watermarking scheme, the discrete wavelet transform/SVD (DWT–SVD) based watermarking scheme, and the SVD was watermarking in the homomorphic domain watermarking scheme. The obtained findings demonstrated the superiority of the KL–DHT-oriented spiral scan watermarking system over the other watermarking methods in the presence of multiple image threats.

Mengchao et al. [18] proposed a high-efficiency one-step single-pixel imaging approach based on the discrete Hartley transform to increase imaging efficiency. The proposed approach requires a real-number computation and does not require a huge number of fringe designs. At the same sampling rate, the number of fringe patterns required for the proposed approach is half that of the four-step phase-shift Fourier technique. Even though it is a single-step approach, it incorporates the concept of differential readings as well as upsampling processing algorithms to increase the restored image's signal-to-noise ratios. When the sampling rate is set to 30%, the peak signal-to-noise ratio and structural similarity index of the recovered target scene reach 20 dB and 80 percent, respectively. It merely takes milliseconds to project such patterns into the target after implying the grey stripe pattern together into a binary pattern. Considering noise interference dramatically, overall experimental findings of the proposed approach are much better than those of the two-step phase-shift method. This approach will represent substantial advancement in the real-time reconstruction of single-pixel imaging, despite the fast introduction of improved technology.

Table 2: Survey on DHT algorithms for image processing

Technique	Advantages	Limitations	Ref.
Multi-image encryption and decryption algorithm	Reduce the transmission load and increase encryption capability	Resist the noise attack only in a certain range	[15]
The blind colour picture watermarking approach is based on the spatial domain	more robustness and can meet the requirement of the 5G network embedding capacity	An inadequate embedding value may cause data extraction to fail.	[16]
KL–DHT-based watermarking algorithm	Provides fidelity and resistance against various image attacks.	System performance depends on the noise density.	[17]
One-step harmonic (OSH) SPI method	High imaging efficiency,	Ambient and projection noise will have an impact on the reconstruction.	[18]
Digital image sharpening method	Improve the quality of picture sharpening by using the fractional-order of derivative.	The sharpened images will be contrasted.	[19]

Chien-Cheng et al. [19] proposed a Riesz transfer function derivative (RFOD) plus discrete Hartley transform approach for digital image sharpening. Image sharpening is well-known in digital image processing for various applications ranging from electronic printing through medical imaging to industry assessment and autonomous guiding in defence applications. Sharpening is used to bring out small details in a picture or to improve elements that were blurred during capture. Sharpening is achieved in the research using spatial filters depending on first or partial derivatives. The Riesz fractional-order derivative and discrete Hartley transform are combined in this research to provide a digital picture sharpening system whose quality may be modified by modifying the numerical technique of the derivative. This digital picture sharpening method uses the RFOD and DHT interpolation with the Mach band effect. The efficiency of the proposed digital picture sharpening technique is proven using many numerical examples. The table 2 describes the DHT algorithms used for image processing.

Compared to other image processing algorithms, spatial domain-based algorithms have a high embedding capacity that can meet the 5G embedded capacity requirement. The key space in this method is large enough, and the security is strong. The next section of this literature survey briefly explains DHT algorithms for image processing.

DHT in Digital Signal Processing

The authors have presented several new signal processing algorithms based on the Discrete Hartley Transform, which are detailed below.

Mahathir et al. [20] proposed a Discrete Hartley Transform-based voice recognition technique. Automated speech recognition is often defined as the capacity to hear and recognize spoken words through converting analogue signals into digital or extracting distinctive voice features such as pitch, frequency, tone, and rhythm to create speaker models and sound samples. This voice register is a segmentation of the human voice's region depending on the origin of the sound, the impression of resonance space, form, colour, sound timbre, and the lower and higher tone generated. Transformation is a conversion applied to a sound sample before it is categorized. When translating voice register signals, the filters can only categorize with a true positive of 69.67 percent on average. Since this sound frequency employed is under ideal circumstances, no noise still influences the classification results of the voice register. The addition of a filter does not affect the classification performance.

Mishal et al. [21] presented a different receiver structure that allows coupling to improve the DHT-OFDM system's bit error rate (BER) quality by enhancing the diversity gain among subcarriers. In DHT-based orthogonal frequency-division multiplexing (OFDM) systems, this coupling between symmetrical carriers is utilized by inserting a frequency-domain subcarrier diversification receiver. In terms of average BER and peak-to-average power ratio, the proposed system has been compared with traditional discrete Fourier transform OFDM (DFT-OFDM), extended DHT-OFDM, and DHT-pre-coded OFDM systems (PAPR). At an average BER of 105, the proposed DHT-OFDM system beats the generalized DHT-OFDM system by 12 dB and the traditional DFT-OFDM and DHT-pre-coded OFDM systems by 17 db. Compared to the DHT-pre-coded OFDM, the traditional DFT-OFDM, and the generalized DHT-OFDM technologies, the proposed DHT-OFDM system has a PAPR increase of roughly 5.5, 2, and 1 dB, correspondingly.

Xing et al. [22] describe a numerically simple and low execution technique for discrete Hartley transform (DHT) pre-coded OFDM systems. Harmonic nulls of frequency-selective decaying channels plague orthogonal frequency-division multiplexing (OFDM). Linear pre-coded (LP-) OFDM is a useful technique for ensuring symbol detection by distributing frequency-domain signals across the entire spectrum. In contrast to traditional DHT-OFDM systems, at the transmitter, both the DHT and the inverse discrete Fourier transform are substituted by a one-level butterfly architecture that generates the time-domain DHTOFDM signal with just one addition per symbol. In contrast to typical DHT-OFDM, where both the DHT and the DFT are necessary to restore the damaged signal with such a single-tap equalization, just the DHT is essential at the receiver to retrieve the distorted signal with such a single-tap equalizer. The theoretical foundation of DHT-OFDM with linear equalizers is provided, and numerical simulation confirms that the proposed DHT-OFDM system has achieved comparable performance to previous LP-OFDMs while reducing implementation complexity peak-to-average average power.

Table 3. Survey on DHT Algorithms for Signal Processing

Technique	Advantages	Limitations	Ref.
Discrete Hartley transform	Female voice recognition	It is only suitable if the sound frequency is in ideal conditions	[20]
Orthogonal frequency-division multiplexing (OFDM) with DHT	Reduce the average BER of the system	PAPR level is high	[21]
low-complexity DHT pre-coded OFDM system	It has a substantially lower PAPR and a lower signal processing complexity.	After equalization, the precoding matrix will influence the noise dispersion on every sub-channel.	[22]
2N-length discrete Hartley transform	Reduce signal processing complexity	Deep fading affects several of the sub-channels.	[23]
Acoustic classification approach	High classification accuracy	Some similar scenes can be misclassified	[24]

Madhan et al. [23] introduced a new very large-scale integration (VLSI) technique for a 2N-length discrete Hartley transform (DHT) which can be efficiently implemented on a highly modular as well as parallel VLSI architecture with a stable structure. This DHT algorithm may be effectively divided into multiple parallel pieces that can be run concurrently. By using the benefits of the proposed method and the reality that we can economically share the multipliers with the same constants, the quantity of multipliers has indeed been greatly decreased, resulting in a very small number of multipliers when compared to the existing techniques. By applying various strategies, the cost and power of the architecture may decrease in both the effective execution of transforms and the reduction/removal of intermediate stages. Furthermore, the proposed approach is well-suited for subexpression sharing techniques, which may be utilized to minimize the hardware complexity for massively parallel VLSI implementations while also increasing the performance of parallel multipliers.

Hitham et al. [24] propose a new discrete Hartley transform-based technique for acoustic environment categorization. The method uses a Hidden Markov Prototype classifier using test data made up of audio clips that determine the environment in which those audio snippets are. The method employs discrete Hartley transform characteristics to provide a collection of characteristics that only need real mathematical computations. As a result, the method may be superior in terms of convenience and computing speed. Experimental results demonstrate that the proposed technique outperforms other initially proposed methods and that using the discrete Hartley transform increases classification performance. Table 3 shows the survey of DHT algorithms used in Signal Processing.

Compared to other image processing algorithms, orthogonal frequency division multiplexing with DHT performs better. The proposed DHT-OFDM system outperforms the generalized DHT-OFDM system by 12 db.

DISCUSSION AND CHALLENGES

Some of the challenges while employing the DHT algorithms for VLSI implementations, signal processing, and image processing applications were discussed in this section.

DHT algorithms for VLSI implementation

The Hartley transform is an integral transform, which is closely related to the Fast Fourier Transform but converts real-valued functions into real-valued ones. The Discrete Hartley Transform (DHT) is a transform that converts data from the time domain to the frequency domain only using real values. Many solutions have been researched to lower the DHT system's latency and area. Systolic algorithms offer the finest VLSI implementations employing pipeline designs for fairly short-length DHT. However, due to so-called retrograde indexing, the present fast DHT algorithms will not have an effective pipeline implementation. The paper presents a parallel VLSI implementation that is both efficient and effective. Because the speed capabilities may be raised with a modest increase in hardware complexity, such pipeline VLSI architecture is a cost-effective option. As a result, new algorithms suitable for a pipeline VLSI execution are required for fast DHT methods. The Dual Split-Radix DHT is a novel split-radix algorithm with high structural qualities, including modularity, regularity, and a basic Signal Flow Graph (SFG) that eliminates retrograde indexing and permits a DHT pipeline execution that is cost-effective. Owing to their uneven computational structure and, in particular, the so-called retrograde indexing, current 1-D radix-2 or split-radix algorithms are challenging to implement utilizing a pipeline VLSI design. VLSI implementation often employs the Dual Split-Radix DHT algorithm, DHT Algorithm using Urdhwa Multiplier, Band convolution, and Short Pseudo cycle convolution DHT methods. We can achieve minimal latency, lower hardware complications, high data transfer speed, low I/O cost, and high processing speed by using DHT techniques.

DHT algorithms for image processing

DHT is utilized in various applications, including image processing, space research, and related technologies. The two most important elements to consider are the amount of delay offered and the amount of space required by the device. The proposed multi-encryption system is based on the quaternion discrete fractional Hartley transform (QDFrHT) and better pixel adaptive diffusion that can boost encryption capacity while lowering key consumption. A spatial domain-oriented blind watermarking system, hybrid Karhunen Loeve transform (KLT), and discrete Hartley transforms (DHT) oriented image watermarking is indeed introduced in the field of image processing. Authors for high imaging efficiency proposed multi-image encryption and decryption algorithms as well as spatial domain-based DHT algorithms. And embedding capacity. A Riesz transfer function derivative (RFOD) plus discrete Hartley transform approach for digital image sharpening was proposed.

DHT algorithm for Signal processing

In the realm of signal processing, a Discrete Hartley Transform-based voice recognition technique is proposed. Transformation is a conversion applied to a sound sample before it is categorized. When translating voice register signals, the filters can only categorize with a true positive of 69.67 percent on average. Orthogonal frequency-division multiplexing (OFDM) with DHT is proposed to reduce the hardware complexity of the system. In DHT-based orthogonal frequency-division multiplexing (OFDM) systems, this coupling between symmetrical carriers is utilized by inserting a frequency-domain subcarrier diversification receiver. An efficient acoustic classification based on DHT is created for better classification. The future directions are explained in the next section.

Future Directions

Even though writers have developed several DHT methods for image and signal processing and VLSI implementations, there are certain restrictions. We can develop a novel approach simply by solving those constraints. The following are suggestions for overcoming some of the common limitations which will be helpful for further research.

- Different complete adders with a set number of transistors were imposed to deal with the challenges of the multiplier.
- The discrete Hartley transform (DHT) eliminates the need for complex mathematics and its inverse.
- Dual Split-Radix DHT has strong structural qualities, including modularity, uniformity, and a simple Signal Flow Graph (SFG) that eliminates retrograde indexing and permits a cost-effective DHT pipeline design.
- A compressor is a necessary tool for rapid multiplication and addition techniques that require a fast processor and little space.

These future directions will be helpful for authors to create a novel and cost-effective technique without limitations.

Conclusion

This review briefly discussed the DHT algorithms developed for image processing, digital signal processing, and VLSI implementations. Also, the limitations while employing this existing DHT algorithm were reviewed in this paper. The most common limitations of the existing techniques are retrograde indexing, hardware complexity, less processing speed, and more space. Some recommendations were made to get around this restriction, enabling the researchers to develop new methods.

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