



## Steady State availability analysis of a Physical System: Sole lasting system of Shoe Industry

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### ABSTRACT

This paper deals with the performance analysis of sole lasting system of a shoe industry using Markovian approach. The plant is divided into many sections like shoe upper manufacturing system, sole lasting system and sole pasting system. In the present work sole lasting system has been taken for performance analysis. The system consists of six subsystems namely, Toe Humidifier Machine, Toe Lasting Machine, Heel Humidifier Machine, Heel Lasting Machine, Heating Chamber Machine and Rubbing & Buffing machine. Failure and repair rates of these subsystems are assumed to be constant and exponentially distributed. A mathematical model pertaining to the real environment of shoe industry has been developed using Markov birth-death process. The differential equations have been derived on the basis of probabilistic approach using transition diagram. These equations are solved using normalizing conditions and recursive method to derive out the steady state availability expression of the system i.e. system's performance criterion. The results give system availability for different combinations of failures and repair rates for various subsystems.

*Keywords: Steady State Availability, Shoe industry, Sole Lasting System, Markov approach.*

### INTRODUCTION

In today's competitive environment, the main concern of the industrialist is to run their industries/manufacturing system failure free for the maximum possible duration to meet the customer's requirement. This fact has motivated for the automation of the industrial system. Although the automation of these systems makes them more sophisticated and also helps in increasing the productivity, but simultaneously the complexity of the system gets increased and hence the risk of failure. The optimum availability analysis is desirable for long working duration with good performance level of the systems in the industries to reduce

the production cost and increase the productivity in this cut throat competition era. So, the reliability engineering is an important tool to compute and improve the systems performance which is widely used now days.

The mechanical systems have attracted the attention of several researchers in the area of reliability engineering. Azaron et al<sup>1</sup> developed a new methodology for reliability evaluation and optimization of non-repairable dissimilar component cold standby redundant systems. Kumar et al<sup>2</sup> derived the availability and mean time to failure (MTTF) expression of washing system of a paper industry using simple probability consideration. Coit et al<sup>3</sup> proposed a multiple objective formulation for maximizing the system availability. Dai et al<sup>4</sup> developed an optimization model for the grid service allocation using Genetic Algorithm. Garg et al<sup>5</sup> developed a reliability model of a block-board manufacturing system of a plywood industry using time dependent and steady state availability under idealized and faulty Preventive Maintenance. Sachdeva et al<sup>6</sup> described a new multi criteria optimization framework for deriving optimal maintenance schedules for preventive maintenance which considers availability, maintenance cost and life cycle costs as the criteria for optimization using Petri Net. Kumar et al<sup>7</sup> discussed the performance evaluation and availability analysis of ammonia synthesis unit of a fertilizer plant. This unit consists of five subunits arranged in series and parallel configurations. For the evaluation of performance and

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analysis of availability, a performance evaluating model has been developed with the help of mathematical formulation based on Markov birth-death process.

Garg et al<sup>8</sup> developed the mathematical model of a cattle feed plant using a Markov birth-death process. The differential equations have been solved for the steady-state. The system performance has also been studied. Kumar and Tewari<sup>9</sup> discussed the mathematical modeling and performance optimization of CO<sub>2</sub> cooling system of a fertilizer plant using genetic algorithm. The differential equations have been derived based on Markov birth-death process using probabilistic approach. These equations are then solved using normalizing conditions to determine the steady state availability of the CO<sub>2</sub> cooling system. Singh I.P.<sup>10</sup> studied the reliability analysis of a complex system having four types of components with pre-emptive priority repairs. Singh and Dayal<sup>11</sup> also discussed the reliability analysis of a repairable system in a fluctuating environment.

Most of the researchers had confined their research work to power plants, sugar industries, chemical industries, paper industries, beverage industries. But the quality work is lacking in the shoe manufacturing industries. Therefore Availability analysis of shoe industry has been chosen for the present study. Here, performance analysis of sole lasting system has been discussed.

### SYSTEM DESCRIPTION

The process flow diagram of sole lasting system of shoe industry is shown in figure no.1. It consists of six subsystems as described below:

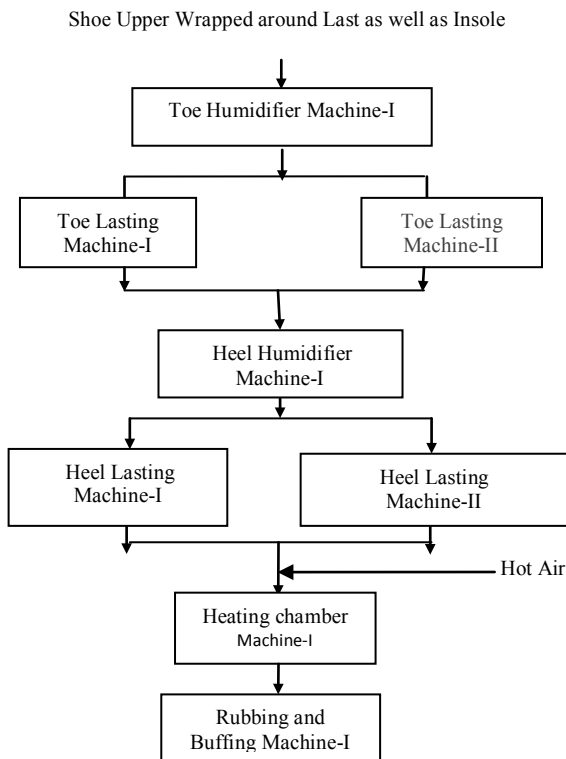


Figure 1. Process Flow Diagram

### ASSUMPTIONS

- i. Failure and repair rates for each subsystem are constant and statistically independent.
- ii. Not more than one failure occurs at a time.
- iii. Performance wise a repaired unit is as good as new.
- iv. The standby units are of the same nature and capacity as the active units.
- v. All the units are initially operating and are in working state.
- vi. Each unit has three states viz. working, degraded and failed.

### NOTATIONS

Subsystem-X<sub>1</sub>: Consists of One Toe-Humidifier Machine subjected to major failure only.

Subsystem-X<sub>2</sub>: Consists of Two Toe-Lasting machines working in parallel subjected to minor as well as major failure only.

Subsystem- X<sub>3</sub>: Consists of One Heel-Humidifier Machine subjected to major failure only.

Subsystem-X<sub>4</sub>: Consists of Two Heel-Lasting machines working in parallel subjected to minor as well as major failure only.

Subsystem- X<sub>5</sub>: Consists of One Heating Chamber subjected to major failure only.

Subsystem-X<sub>6</sub>: Consists of One Rubbing and Buffing Machine subjected to major failure only.

Superscript ‘o’: Subsystems in operating state.

Superscript ‘g’: Subsystems in good but not in operating state.

Superscript ‘r’: Subsystems in under repair.

Superscript ‘qr’: Subsystems in queuing for repair.

$\lambda_i, \lambda_{3-i}$ : Failure rates Toe Lasting Machines (X<sub>2i</sub>and X<sub>2i</sub>)

$\lambda_j, \lambda_{3-j}$ : Failure rates Heel Lasting Machines (X<sub>4j</sub>and X<sub>4j</sub>)

$\lambda_1, \lambda_2, \lambda_3, \lambda_4$ : Failure rates of X<sub>1</sub>, X<sub>3</sub>, X<sub>5</sub> and X<sub>6</sub>.

$\lambda_7, \lambda_8$ : Failure rates of X<sub>2</sub> & X<sub>4</sub> from in reduced state

$\mu_i, \mu_{3-i}$ : Repair rates of (X<sub>2i</sub>and X<sub>2i</sub>)

$\mu_j, \mu_{3-j}$ : Repair rates of (X<sub>4j</sub>and X<sub>4j</sub>)

$\mu_1, \mu_2, \mu_3, \mu_4$ : Repair rates of X<sub>1</sub>, X<sub>3</sub>, X<sub>5</sub> and X<sub>6</sub>.

$\mu_7, \mu_8$ : Repair rates of X<sub>2</sub> & X<sub>4</sub> In reduced State.

$P_i(t)$ : Probability that at time ‘t’ all units are good and the system is in i<sup>th</sup> state.

’ : Derivatives w.r.t. ‘t’

Based on above assumptions and notations the state transition diagram of sole lasting system has been developed as shown in Figure 1.

### MATHEMATICAL MODELING OF THE SYSTEM

The differential equations associated with the transition diagram shown in figure no. 2, Appendix ‘A’ are developed on the basis of Markov birth-death process. Various probability considerations generate the following sets of differential equations:

$$P_1'(t) + T_1 P_1(t) = \mu_1 P_5(t) + \mu_3 P_6(t) + \mu_5 P_7(t) + \mu_6 P_8(t) + \mu_2 P_2(t) + \mu_4 P_3(t) + \mu_7 P_{13}(t) + \mu_8 P_{18}(t) \quad (1)$$

$$P_2'(t) + T_2P_2(t) = \mu_1P_9(t) + \mu_3P_{10}(t) + \mu_5P_{11}(t) + \mu_6P_{12}(t) + \mu_7P_{23}(t) + \lambda_2P_1(t) \quad (2)$$

$$P_3'(t) + T_3P_3(t) = \mu_3P_{15}(t) + \mu_5P_{16}(t) + \mu_6P_{17}(t) + \mu_8P_{24}(t) + \mu_1P_{14}(t) + \lambda_4P_1(t) \quad (3)$$

$$P_4'(t) + T_4P_4(t) = \mu_1P_{19}(t) + \mu_3P_{20}(t) + \mu_5P_{21}(t) + \mu_7P_{19}(t) + \mu_6P_{23}(t) + \lambda_2P_3(t) \quad (4)$$

$$P_i'(t) + \mu_jP_i(t) = \lambda_jP_k(t) \quad (5)$$

(For k=1, j=1, 3, 5, 6; i=5, 6, 7, 8 respectively)  
 (For k=2, j=1, 3, 5, 6, 7; i=9, 10, 11, 12, 13 respectively)  
 (For k=3, j=1, 3, 5, 6, 8; i=14, 15, 16, 17, 18 respectively)  
 (For k=4, j=1, 3, 5, 6, 7, 8; i=19, 20, 21, 22, 23, 24 respectively)

**Where**

$$T_1 = (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6)$$

$$T_2 = (\lambda_1 + \lambda_3 + \mu_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7)$$

$$T_3 = (\lambda_1 + \lambda_3 + \mu_4 + \lambda_2 + \lambda_5 + \lambda_6 + \lambda_8)$$

$$T_4 = (\lambda_1 + \lambda_3 + \lambda_8 + \lambda_5 + \lambda_6 + \lambda_7)$$

With initial conditions at time t = 0

$$P_i(t) = 1 \text{ for } i=0,$$

$$P_i(t) = 0 \text{ for } i \neq 0 \quad (6)$$

**STEADY STATE BEHAVIOR OF THE SYSTEM**

Steady State analysis i.e. when  $t \rightarrow \infty$  and  $d/dt \rightarrow 0$  applying on set of first order differential equations (1 to 5), we get:

$$T_1P_1 = \mu_1P_5 + \mu_3P_6 + \mu_5P_7 + \mu_6P_8 + \mu_2P_2 + \mu_4P_3 + \mu_7P_{13} + \mu_8P_{18}$$

$$T_2P_2 = \mu_1P_9 + \mu_3P_{10} + \mu_5P_{11} + \mu_6P_{12} + \mu_7P_{23} + \lambda_2P_1$$

$$T_3P_3 = \mu_3P_{15} + \mu_5P_{16} + \mu_6P_{17} + \mu_8P_{24} + \mu_1P_{14} + \lambda_4P_1$$

$$T_4P_4 = \mu_1P_{19} + \mu_3P_{20} + \mu_5P_{21} + \mu_7P_{19} + \mu_6P_{23} + \lambda_2P_3$$

$$\mu_jP_i = \lambda_jP_k$$

(For k=1, j=1, 3, 5, 6; i=5, 6, 7, 8 respectively)  
 (For k=2, j=1, 3, 5, 6, 7; i=9, 10, 11, 12, 13 respectively)  
 (For k=3, j=1, 3, 5, 6, 8; i=14, 15, 16, 17, 18 respectively)  
 (For k=4, j=1, 3, 5, 6, 7, 8; i=19, 20, 21, 22, 23, 24 respectively)

Using Normalizing Condition and initial conditions at time t = 0,  $P_i(t) = 1$  for  $i=0$  and  $P_i(t) = 0$  for  $i \neq 0$  i.e. sum of all the probabilities is equal to one, i.e.

$$\sum_{i=1}^{24} P_i = 1$$

$$P_1 = \left[ \frac{1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} + P_{11} + P_{12} + P_{13} + P_{14} + P_{15} + P_{16} + P_{17} + P_{18} + P_{19} + P_{20} + P_{21} + P_{22} + P_{23} + P_{24}}{1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} + P_{11} + P_{12} + P_{13} + P_{14} + P_{15} + P_{16} + P_{17} + P_{18} + P_{19} + P_{20} + P_{21} + P_{22} + P_{23} + P_{24}} \right]^{-1}$$

The Steady State Availability of the system  $A_{ss}$  is given by

$$A_{ss} = P_1 + P_2 + P_3 + P_4 = P_1(1 + M_5 + M_4 + M_3)$$

**PERFORMANCE ANALYSIS**

The effects of failure and repair rates of various subsystems comprising the system are examined and their impact on system availability is shown in the following tables:

Table 1: Effect of Failure and Repair Rate of Toe Humidifier Machine on System Availability

$\lambda_1$ $\mu_1$	0.00138	0.00238	0.00338	0.00438	Other Constant Parameters
0.025	0.8646	0.8357	0.8086	0.7832	$\lambda_2=0.0001, \mu_2=0.001, \lambda_3=0.00166, \mu_3=0.03, \lambda_4=0.002, \mu_4=0.02, \lambda_5=0.0008, \mu_5=0.05, \lambda_6=0.0001, \mu_6=0.01, \lambda_7=0.0001, \mu_7=0.001, \lambda_8=0.002, \mu_8=0.02.$
0.035	0.8766	0.8551	0.8347	0.8152	
0.045	0.8834	0.8664	0.8500	0.8342	
0.055	0.8878	0.8737	0.8600	0.8467	

Table 2: Effect of Failure and Repair Rate of Toe Lasting Machine on System Availability

$\lambda_2 = \lambda_7$ $\mu_2 = \mu_7$	0.0001	0.0002	0.0003	0.0004	Other Constant Parameters
0.001	0.8646	0.8513	0.8349	0.8164	$\lambda_1=0.00138, \mu_1=0.025, \lambda_3=0.00166, \mu_3=0.03, \lambda_4=0.002, \mu_4=0.02, \lambda_5=0.0008, \mu_5=0.05, \lambda_6=0.0001, \mu_6=0.01, \lambda_8=0.002, \mu_8=0.02$
0.002	0.8675	0.8609	0.8536	0.8458	
0.003	0.8684	0.8536	0.8589	0.8543	
0.004	0.8688	0.8647	0.8611	0.8577	

Table 3: Effect of Failure and Repair rates of Heel Humidifier Machine on System Availability

$\lambda_3$ $\mu_3$	0.00166	0.00266	0.00366	0.00466	Other Constant Parameters
0.03	0.8646	0.8404	0.8174	0.7957	$\lambda_1=0.00138, \mu_1=0.025, \lambda_2=0.0001, \mu_2=0.001, \lambda_4=0.002, \mu_4=0.02, \lambda_5=0.0008, \mu_5=0.05, \lambda_6=0.0001, \mu_6=0.01, \lambda_7=0.0001, \mu_7=0.001, \lambda_8=0.002, \mu_8=0.02$
0.04	0.8751	0.8563	0.8383	0.8211	
0.05	0.8815	0.8662	0.8514	0.8372	
0.06	0.8859	0.8730	0.8604	0.8482	

Table 4: Effect of Failure and Repair Rate of Heel Lasting Machine on System Availability

$\lambda_4 = \lambda_8$	0.002	0.004	0.006	0.008	Other Constant Parameters
$\mu_4 = \mu_8$					
0.02	0.8646	0.8584	0.8506	0.8417	$\lambda_1=0.00138,$ $\mu_1=0.025,$ $\lambda_2=0.0001,$ $\mu_2=0.001,$ $\lambda_3=0.00166,$ $\mu_3=0.03,$ $\lambda_5=0.0008,$ $\mu_5=0.05,$ $\lambda_6=0.0001,$ $\mu_6=0.01,$ $\lambda_7=0.0001,$ $\mu_7=0.001$
0.03	0.8696	0.8667	0.8628	0.8582	
0.04	0.8718	0.8702	0.8680	0.8652	
0.05	0.8729	0.8720	0.8706	0.8688	

Table 5: Effect of Failure and Repair rates of heating Chamber Machine on System Availability

$\lambda_5$	0.0008	0.0009	0.0010	0.0011	Other Constant Parameters
$\mu_5$					
0.05	0.8646	0.8631	0.8616	0.8602	$\lambda_1=0.00138, \mu_1=0.025,$ $\lambda_2=0.0001, \mu_2=0.001,$ $\lambda_3=0.00166, \mu_3=0.03,$ $\lambda_4=0.002, \mu_4=0.02,$ $\lambda_6=0.0001, \mu_6=0.01,$ $\lambda_7=0.0001, \mu_7=0.001$ $\lambda_8=0.002, \mu_8=0.02$
0.06	0.8666	0.8654	0.8641	0.8629	
0.07	0.8681	0.8670	0.8659	0.8648	
0.08	0.8692	0.8682	0.8673	0.8663	

Table 6: Effect of Failure and Repair rates of Rubbing and Buffing Machine on System Availability

$\lambda_6$	.0001	.00011	.00012	.00013	Other Constant Parameters
$\mu_6$					
.01	.8646	.8639	.8631	.8624	$\lambda_1=0.00138, \mu_1=0.025,$ $\lambda_2=0.0001, \mu_2=0.001,$ $\lambda_3=0.00166, \mu_3=0.03,$ $\lambda_5=0.0008, \mu_5=0.05,$ $\lambda_4=0.002, \mu_4=0.02,$ $\lambda_7=0.0001, \mu_7=0.001$ $\lambda_8=0.002, \mu_8=0.02$
.0115	.8656	.8650	.8643	.8637	
.0140	.8668	.86624	.8657	.8652	
.0165	.8676	.8671	.8667	.86622	

**RESULT AND DISCUSSION**

From table 1 to 6, it has been observed that the increase in failure and repair rates of various subsystems affects the availability of the system and need to be addressed.

Table 1 shows the effect of failure and repair rate of toe-humidifier machine on the steady state availability of sole lasting system, as the failure rate ( $\lambda_1$ ) increases from 0.00138 to 0.00438 the system’s availability reduces considerably by 9.41%. Similarly as the repair rate ( $\mu_1$ ) increases from 0.025 to 0.055, the unit availability increases from 2.68% to 8.1%.

Table 2 reveals the effect of failure and repair rates of Toe-Lasting machine on the availability of Sole Lasting system, as the failure rate ( $\lambda_2 = \lambda_7$ ) increases from 0.0001 to 0.0004 the system availability reduces by 5.574%. Similarly as the repair rate ( $\mu_2 = \mu_7$ ) increases from 0.001 to 0.004, the system’s availability hardly increases from 0.485% to 5.058%.

Table 3 depicts the effect of failure and repair rates of the Heel Humidifier machine on the availability of the sole lasting system, as the failure rate ( $\lambda_3$ ) of Heel Humidifier machine increases from 0.00166 to 0.00466 the availability decreases by 7.96%. Similarly as the repair rate ( $\mu_3$ ) increases from 0.03 to 0.06 the availability increases from 2.46% to 6.59%.

Table 4 highlights the effect of failure and repair rates of the Heel Lasting machine on the availability of the sole lasting system, as the failure rate ( $\lambda_4 = \lambda_8$ ) of skiving machine increases from .002 to .008 the availability decreases by 2.64%. Similarly as the repair rate ( $\mu_4 = \mu_8$ ) increases from .020 to .50 the system availability increases from 0.95% to 3.21%.

Table 5 explains the effect of failure and repair rates of the Heating Chamber machine on the availability of the sole lasting system, as the failure rate ( $\lambda_5$ ) of Heating Chamber machine increases from 0.0008 to 0.011 the availability decreases by 0.508%. Similarly as the repair rate ( $\mu_5$ ) increases from 0.05 to 0.08 the system’s availability increases from 0.532% to 0.709%.

Table 6 reveals the effect of failure and repair rates of the Rubbing and Buffing machine on the availability of the sole lasting unit, as the failure rate ( $\lambda_6$ ) of Rubbing and Buffing machine increases from 0.0001 to 0.00013 the availability decreases by 0.254%. Similarly as the repair rate ( $\mu_6$ ) increases from 0.020 to 0.080 the unit availability increases from 0.346% to 0.439%.

**CONCLUSION**

The steady state analysis of sole lasting system of shoe Industry has been carried out with the help of mathematical modeling using probabilistic approach. The results are shown in tables 1 to 6 are derived to assists the maintenance decisions where repair priorities should be given to subsystems of sole lasting unit. Table 1 clearly specifies that the toe humidifier machine is the most critical subsystem as far as maintenance aspect is concerned and given top priority. The heel humidifier machine should be given second priority as the effect of its failure and repair rate on the system performance is much higher than that of other machines. Therefore, on the basis of above performance analysis, the maintenance priorities should be given as per following order:

Subsystem	Failure rate	%age decrease in steady state availability	Repair rate	%age increase in steady state availability	Suggested repair priorities
Toe Humidifier Machine	0.00138 to 0.00438	9.414	0.025 to 0.055	2.68 to 8.107	I
Toe Lasting Machines	0.0001 to 0.0004	5.574	0.001 to 0.004	0.48 to 5.05	III
Heel Humidifier Machine	0.00166 to 0.00466	7.969	0.03 to 0.06	2.46 to 6.597	II
Heel Lasting Machine	0.002 to 0.008	2.648	0.02 to 0.05	0.95 to 3.21	IV
Heating Chamber	0.0008 to 0.0011	0.508	0.05 to 0.08	0.53 to 0.70	V
Rubbing and Buffing Machine	0.0001 to 0.00013	0.254	0.01 to 0.0165	0.34 to 0.439	VI

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