J. Integr. Sci. Technol. 2022, 10(3), 193-197 Article

Journal of Integrated
SCIENCE & TECHNOLOGY

An EOQ model for Inventory System dependent upon on hand inventory without shortages

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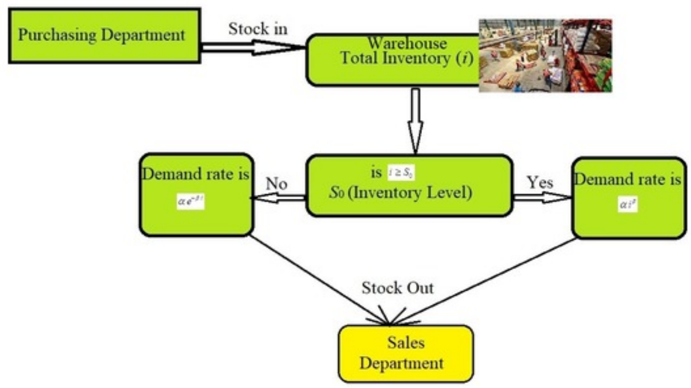
Received on: 11-Jun-2022, Accepted on: 28-Aug-2022 and Published on: 31-Aug-2022

ABSTRACT

The inventory management in supply chain remain crucial for the proper availability of all goods. The mathematical and computer algorithms are implemented to channelize the distribution and availability of stock at particular time and specific point of chain supply. The cumulative impact of all algorithms and models describes the physical stock availability under different situations arising because of high demand, variable stock, distributed inventory and all other alike factors. In this paper we present a model to study a situation where the demand rate declines along with stock level down. The demand rate is different for different situations i.e, the demand rate is αi^{β} when $i \geq s_0$ and $\alpha e^{-\beta i}$ when $0 \leq i \leq s_0$ where i is the inventory level. Numerical examples and sensitivity analysis are presented to illustrate the model developed.



Keywords: Inventory, demand and storage, mathematical algorithms, supply algorithms, Economic Order Quantity

INTRODUCTION

The inventory management algorithms and model have been crucial in proper management of large stocks with the proper supply at the end point. To meet the future demand most of the existing inventory models in the literature assume that items can be stored indefinitely. However, in the course of time certain types of commodities either deteriorate or become obsolete. Deterioration is also known as damage or spoilage in inventory models is now of immense practical importance, which is gaining attention from the researchers. Deterioration occurs with passage of time depending upon the kind of items considered. For example: food stuffs, alcohol, vegetables, meat, photographic films, etc, where deterioration is usually observed during their normal storage period. As reported by Levin et al (1972)[12] and Silver and Peterson (1985),[25] sales at the retail level tend to be proportional to inventory displayed and a large piles of goods displayed in a supermarket will lead the customers to bye more. These observations have attracted many marketing researchers and practioners to investigate the modeling aspects of this phenomenon. To minimize the cost with the assumption that stock- dependent consumption rate is a function of the initial stock level Gupta and Vrat (1986)[8] first developed a model for consumption environment. Silver and Meal (1973),[24] Datta and Pal (1988)[4] has progressed the model where demand rate is not required to be constant.

Covert and Philip (1973),[3] Giri et al. (2003),[6] Ghosh and Chaudhari (2004),[5] Sana et. al. (2004)[21] are developed lot size models for deteriorating items. Mishra and Tripathy (2010),[26] Kawale and Bansode (2012), etc., considered model saving deterioration rate proportional to time. A Price and ramp-type demand which also depends on time has been developed by Wang, Chuanxu, Huang, Rongbing (2014).[27] Patro et. al. (2017) & (2018)[14,15] developed Economic Order Quantity (EOQ) models without deterioration and with deterioration using allowable proportionate discount under learning effects respectively. A more practical and realistic EOQ model is the one considering items to be imperfect. Porteus (1986),[17] Rosenblatt and Lee (1987),[11] Raouf,

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Cite as: *J. Integr. Sci. Technol., 2022, 10(3), 193-197.*

 ISSN: 2321-4635 http://pubs.iscience.in/jist

Jain, and Sathe (1983)[18] are few researchers who studied the basic EOQ model for influence of defective items. It is supposed that, there is no fault in the screening process of traditional inventory models that identifies the defective items, the items are screened without any inspection, i.e. zero error inspection is carried out. But in 2000 Salameh and Jaber eveloped model with considering after hundred % screening the imperfect quality items collect a single batch and then sold.Similar work was done by Goyal and Cardenas-Barron (2002).[7] Nita Shah (2012)[23] developed a time-proportional deterioration model without shortages and with replenishment policy for items having demands depending on price. Considering selling price dependent demand Sarkar (2013)[22] developed a deteriorating model. For deteriorating items Chowdhury and Ghosh (2014)[2] developed an inventory model with price and stock sensitive demand. Khana et. al. (2017)[10] considering price dependent demand, developed a lot size deteriorating model for imperfect quality items. Uncertainties in some situations is due to fuzziness was primarily introduced by Zadeh, also some strategies for decision making in fuzzy environment was proposed by Zadeh et. al (1970).[28] Pattnaik et al (2021)[16] also worked on linearly deteriorating EOQ model for imperfect items with price dependent demand under different fuzzy environments.

Among the important paper published so far with inventory-level-dependent demand rate, mention may be made of by Gupta and Vrat (1986),[8] Mandal and Phaujdar (1989),[13] baker and Urban (1988)[1] etc. Gupta and Vrat (1986)[8] have discussd a situation where the demand rate has been assumed to dependent on the order quantity whereas, Mandal and Phaujdar (1989)[13] have discussed an inventory level. Baker and Urban (1988)[1] have analyzed a similar situation assuming the demand rate to be dependent on the on-hand inventory i according to the relation $R(i) = \alpha i^{\beta}$ where $\alpha > 0$, $0 < \beta < 1$. Sahu et al (2007)[19] have analysed a similar situation, assuming the demand rate to be depend on the on-hand inventory i according to the relation $R(i) = \alpha e^{-\beta i}$ where $\alpha > 0$, $0 < \beta < 1$.

The present inventory model makes an attempt to study the situation where the demand rate decline along with stock level down to a certain level of inventory with demand rate αi^{β} and also the demand rate become $\alpha e^{-\beta i}$ for the rest of the cycle. It is well known that the stock level has a motivation affect on the customers but the experience shows that customers arrive to purchase goods attracted by huge stock down to a certain level of declining inventory. Only a limit numbers of customer arrive to purchase goods owing to such factors goodwill, good quantity, genuine price level of the goods, locality of the shop, good quality of the items etc.

Fundamental assumptions and notations of the Model

The basic assumption made to develop the proposed model are the following.

(1)Replenishment rate is infinite i.e, replenishment is instantaneous but replenishment size is finite.

(2) Lead time is zero.

(3) No shortages are permitted.

(4) The selling price s, unit cost C, holding cost C_1per unit per unit time and replenishment cost C_3 per replenishment are known and constant.

(5) The time horizon is finite.

(6) Only one item involves in the inventory system.

(7) The demand rate is dependent on the on-hand inventory down to a level S_0, beyond Which, it is assumed to be constant. The demand rate $R(i)$ of the item when the on hand inventory level is

$$R(i) = \alpha i^{\beta}, \qquad i \geq S_0$$

$$R(i) = \alpha e^{-\beta i}, \quad 0 \leq i \leq S_0, \quad \alpha > 0, \quad 0 < \beta < 1,$$

Basics Governing equation and their solution

The basic equations governing the present model are the following:

$$\frac{di}{dt} = -\alpha i^{\beta}, \qquad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{di}{dt} = -\alpha e^{-\beta i}, \; t_1 \leq t \leq T \tag{2}$$

Where $t, i, \alpha, \beta, t_1, T$ are defined

The solution of the differential equation (1) using the initial condition $i = S$ at $t = 0$ is

$$i = \left\{-\alpha\left(1-\beta\right)t + s^{1-\beta}\right\}^{\frac{1}{1-\beta}}, 0 \leq t \leq t_1 \tag{3}$$

Using the condition $i = S_0$ at $t = t_1$ is,

$$S_0 = \left\{-\alpha\left(1-\beta\right)t_1 + s^{1-\beta}\right\}^{\frac{1}{1-\beta}} \tag{4}$$

$$\text{So, } t_1 = \frac{S^{1-\beta} - {S_0}^{1-\beta}}{\alpha(1-\beta)} \tag{5}$$

The solution of the differential equation (2) using the condition $i = 0$ at $t = T$, is

$$i = \frac{\ln\left(1+\alpha\beta(T-t)\right)}{\beta}, t_1 \leq t \leq T \tag{6}$$

Using the condition $i = S_0$ at $t = t_1$, we find from (6)

$$S_0 = \frac{\ln\left(1+\alpha\beta\left(T-t_1\right)\right)}{\beta}, T = t_1 + \frac{e^{\beta S_0} - 1}{\alpha\beta} \tag{7}$$

From (5) and (7) we get

$$T = \frac{S^{1-\beta} - S_0^{1-\beta}}{\alpha\left(1-\beta\right)} + \frac{e^{\beta S_0} - 1}{\alpha\beta} \tag{8}$$

$$\text{Now, } H = \int_0^T i dt = \int_0^{t_1} i dt + \int_{t_1}^T i dt$$

Substituting the value of i in the above integrals and then integrating, we find

$$H = \int_0^{t_1} \left\{-\alpha\left(1-\beta\right)t + S^{1-\beta}\right\}^{\frac{1}{1-\beta}} dt + \int_{t_1}^{T} \frac{\ln\left(1+\alpha\beta(T-t)\right)}{\beta} dt$$

Eliminating t_1 and T

$$H=\frac{\alpha\beta^3\left(S^{2-\beta}-S_0^{2-\beta}\right)+\left(2-\beta\right)\left(\alpha\beta^2S_0+\alpha\beta+1\right)\left(e^{\beta S_0}-1\right)}{\alpha^2\beta^3\left(2-\beta\right)} \tag{9}$$

Now the profit function $\pi(S)$ (profit per unit time) is

$$\pi(S)=\frac{(s-C)S-C_1H-C_3}{T}$$

$$\pi(S)=\frac{(s-C)S-C_1\dfrac{\alpha\beta^3\left(S^{2-\beta}-S_0^{2-\beta}\right)+\left(2-\beta\right)\left(\alpha\beta^2S_0+\alpha\beta+1\right)\left(e^{\beta S_0}-1\right)}{\alpha^2\beta^3\left(2-\beta\right)}-C_3}{\dfrac{S^{1-\beta}-S_0^{1-\beta}}{\alpha(1-\beta)}+\dfrac{e^{\beta S_0}-1}{\alpha\beta}} \tag{10}$$

The necessary condition for $\pi(S)$ to be a maximum is

$\dfrac{d\pi(S)}{dS}=0$ gives

$$\begin{aligned}&S^{2-2\beta}\left\{-C_1\beta(2-\beta)-C_1\alpha\beta(1-\beta)\right\}\\&+S^{1-\beta}\left\{\begin{matrix}\alpha\beta(2-\beta)(s-C)+C_1\beta(2-\beta)S_0^{1-\beta}\\-C_1(1-\beta)(2-\beta)\left(e^{\beta S_0}-1\right)-\alpha\beta(1-\beta)(2-\beta)(s-C)\end{matrix}\right\}\\&+S^{-\beta}\left\{\begin{matrix}C_1\alpha\beta(1-\beta)S_0^{2-\beta}-C_1\alpha\beta(1-\beta)(2-\beta)\\\left(\alpha\beta^2S_0+\alpha\beta+1\right)\left(e^{\beta S_0}-1\right)-C_3\alpha\beta(1-\beta)(2-\beta)\end{matrix}\right\}\\&+\alpha(1-\beta)(2-\beta)(s-C)\left(e^{\beta S_0}-1\right)-\alpha\beta(2-\beta)(s-C)S_0^{1-\beta}\\&=0\end{aligned} \tag{11}$$

$$T=\frac{S^{1-\beta}-S_0^{1-\beta}}{\alpha(1-\beta)}+\frac{e^{\beta S_0}-1}{\alpha\beta} \tag{12}$$

The roots of the equation (11) give the global maximum for the profit function $\pi(S)=\pi(S^*)$ occurs at $S=S^*$ and $T=T^*$. Such a positive root S of equation (11) for which

$\dfrac{d^2\pi(S)}{dS^2}<0$ gives a local maximum for the profit function $\pi(S)$

NUMERICAL EXAMPLES

To illustrate the model developed, the following five numerical examples have been considered.

Example 1: - We choose $\alpha=0.6$, $\beta=0.2$, $s=60$, $C=12$, $S_0=8$, $C_3=10$, $C_1=0.5$, Using the decision rule, we get $S=107.2$, $T=109.63$ years and $\pi(S^*)=\$\ 19.757$ / year.

Example 2: - We choose $\alpha=0.6$, $\beta=0.2$, $s=70$, $C=14$, $S_0=8$, $C_3=10$, $C_1=0.5$, Using the decision rule, we get $S=119.43$, $T=117.55$ years and $\pi(S^*)=\$\ 27.735$/ year.

Example 3: - We choose $\alpha=0.6$, $\beta=0.2$, $s=80$, $C=16$, $S_0=8$, $C_3=10$, $C_1=0.5$, Using the decision rule, we get $S=131.88$, $T=125.44$ years and $\pi(S^*)=\$\ 36.006$/ year.

Example 4: - We choose $\alpha=0.6$, $\beta=0.2$, $s=50$, $C=10$, $S_0=8$, $C_3=10$, $C_1=0.5$, Using the decision rule, we get $S=95.191$, $T=101.68$ years and $\pi(S^*)=\$\ 12.098$ / year.

Example 5: - We choose $\alpha=0.6$, $\beta=0.2$, $s=40$, $C=8$, $S_0=8$, $C_3=10$, $C_1=0.5$, Using the decision rule, we get $S=83.452$, $T=93.71$ years and $\pi(S^*)=\$\ 4.7886$ / year.

SENSITIVITY ANALYSIS

This numerical values are computed by using MATAL

Table.1. Changes in α

α	β	s	C	S_0	C_3	C_1	S	$\pi(S)$	$T(S)$
0.4							92.386	1.277	149.690
0.5							98.694	10.780	124.820
0.6	0.2	60	12	8	10	0.5	107.200	19.757	109.630
0.7							117.090	28.560	99.466
0.8							127.930	37.361	92.214

Interpretation of the Table-I: The inventory (S) and the profit $\pi(S)$ rises with increase in the value of parameter α whereas the time $T(S)$ decreases.

Table. II. Changes in β

α	β	s	C	S_0	C_3	C_1	S	$\pi(S)$	$T(S)$
	0.1						86.797	1.605	97.260
	0.15						100.213	12.780	101.590
0.6	0.2	60	12	8	10	0.5	107.200	19.757	109.630
	0.25						149.030	26.111	126.810
	0.3						225.110	33.701	151.020

Interpretation of the Table-II: The inventory (S), profit $\pi(S)$ and the time $T(S)$ gets increase with increase the value of parameter β.

Table. III. Changes in s

α	β	s	C	S_0	C_3	C_1	S	$\pi(S)$	$T(S)$
		40					77.694	1.275	89.721
		50					92.230	10.237	99.688
0.6	0.2	60	12	8	10	0.5	107.200	19.757	109.630
		70					122.530	29.776	119.520
		80					138.170	40.245	129.370

Interpretation of the Table-III: Similarly, in Table-II, here also the inventory(S), profit $\pi(S)$ and the time $T(S)$ gets increase with increase the value of parameter of selling price s.

Table. IV. Changes in *s*

<table>
<tr><th>α</th><th>β</th><th>s</th><th>C</th><th>S_0</th><th>C_3</th><th>C_1</th><th>S</th><th>$\pi(S)$</th><th>$T(S)$</th></tr>
<tr><td rowspan="5">0.6</td><td rowspan="5">0.2</td><td rowspan="5">0.2</td><td>8</td><td rowspan="5">8</td><td rowspan="5">10</td><td rowspan="5">0.5</td><td>113.290</td><td>23.708</td><td>113.590</td></tr>
<tr><td>10</td><td>110.230</td><td>21.723</td><td>111.610</td></tr>
<tr><td>12</td><td>107.200</td><td>19.757</td><td>109.630</td></tr>
<tr><td>14</td><td>104.170</td><td>17.811</td><td>107.640</td></tr>
<tr><td>16</td><td>101.160</td><td>15.886</td><td>105.660</td></tr>
</table>

Interpretation of the Table-IV: On increase with the unit cost(C), the inventory(S), profit $\pi(S)$ and the time $T(S)$ gets decrease.

Table. V. Changes in replenishment cost C_3

<table>
<tr><th>α</th><th>β</th><th>s</th><th>C</th><th>S_0</th><th>C_3</th><th>C_1</th><th>S</th><th>$\pi(S)$</th><th>$T(S)$</th></tr>
<tr><td rowspan="5">0.6</td><td rowspan="5">0.2</td><td rowspan="5">0.2</td><td rowspan="5">12</td><td rowspan="5">8</td><td>5</td><td rowspan="5">0.5</td><td>107.070</td><td>19.803</td><td>109.540</td></tr>
<tr><td>10</td><td>107.200</td><td>19.757</td><td>109.630</td></tr>
<tr><td>15</td><td>107.320</td><td>19.711</td><td>109.710</td></tr>
<tr><td>20</td><td>107.450</td><td>19.666</td><td>109.790</td></tr>
<tr><td>25</td><td>107.570</td><td>19.620</td><td>109.870</td></tr>
</table>

Interpretation of the Table-V: In the increase of replenishment cost, the inventory(S), profit $\pi(S)$ and the time $T(S)$ increase slowly.

Table. VI. Changes in holding cost C_1

<table>
<tr><th>α</th><th>β</th><th>s</th><th>C</th><th>S_0</th><th>C_3</th><th>C_1</th><th>S</th><th>$\pi(S)$</th><th>$T(S)$</th></tr>
<tr><td rowspan="5">0.6</td><td rowspan="5">0.2</td><td rowspan="5">0.2</td><td rowspan="5">12</td><td rowspan="5">8</td><td rowspan="5">10</td><td>0.3</td><td>157.460</td><td>31.980</td><td>141.210</td></tr>
<tr><td>0.4</td><td>125.690</td><td>25.452</td><td>121.530</td></tr>
<tr><td>0.5</td><td>107.200</td><td>19.757</td><td>109.630</td></tr>
<tr><td>0.6</td><td>95.147</td><td>14.538</td><td>101.650</td></tr>
<tr><td>0.7</td><td>86.699</td><td>9.6171</td><td>95.936</td></tr>
</table>

Interpretation of the Table-VI: On increase with the holding cost (C_1), the inventory (S), profit $\pi(S)$ and the time $T(S)$ gets reduce.

CONCLUSION

The stock level has a motivational effect on the customers, but experience shows that some customers always arrive to purchase goods for other reasons, namely goodwill, genuine price etc.

The current inventory model contains two types of demand rate of an item varies due to stock level. The demand rate is dependent in the initial stock level are analyzed as are models in which demand rate is dependent. On the instantaneous stock level, the stock level has a motivational effect on the customers.

ACKNOWLEDGMENTS

This research article was supported by the department of Electrical Engineering (to use the MATLAB). We thank our colleagues from the School of Science who provided insight and expertise that greatly assisted the research, although they may not agree with all of the interpretations/conclusions of this paper.

CONFLICT OF INTEREST

Authors declare there is no conflict of interest for publication of this work.

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